TW776/876 **Numerical Methods** **2011**

Lecturers:

Lecturers: Prof. JAC Weideman — Dr SJ vd Walt

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Module:

Credits: 16

Time & Place: Mo 9:00-9:50, Tu 12:00-12:50, Th 12:00-12:50; room A409.

Contents: The module focuses on numerical methods for matrix computations. We look at the effective solution of square linear systems, least squares problems, and the eigenvalue problem. We consider direct as well as iterative methods, with special attention to sparse matrices and structured matrices. Pitfalls such as numerical instability and ill-conditioning are pointed out. Model problems are drawn from partial differential equations and image processing. Theory, algorithmic aspects, and applications are emphasised in equal measure. (A tentative list of topics is given on the reverse side.)

Prerequisites: An undergraduate module on matrices/linear algebra plus some computing skills in an environment such as MATLAB or Python.

Text Books:

U Ascher & C Greif, A First Course on Numerical Methods, 2011 (available online to participating students; link on course webpage).

The following reference is available for free online:

Y Saad, Iterative Methods for Sparse Linear Systems, 2nd edition, 2003 (link on the class web page).

In addition, the following two resources are available at the reserve desk of the Engineering Library, for short term borrowing:

LN Trefethen & D Bau, Numerical linear algebra, SIAM, 1997.

J Demmel, Applied numerical linear algebra, SIAM, 1997.

Assessment: Continuous assessment, with the final mark computed as follows

Quarter Test 1	20 %
Quarter Test 2	20 %
Assignments	60 %
	100 %

Pass	50 %
Distinction	75 %

Tentative list of topics

I. Square matrices

- Solution of linear systems: direct vs iterative methods
- Special structure that can be exploited:
  - sparse matrices
  - symmetric positive definite matrices
  - circulant and Toeplitz matrices
- Pitfalls:
  - Ill-conditioning
  - Numerical instability
- Algorithms:
  - Gaussian elimination, Cholesky
  - Jacobi, Gauss-Seidel, SOR
  - conjugate gradients, plus pre-conditioning

II. Rectangular matrices

- Least squares problems
- QR and SVD decomposition
- Algorithms:
  - Householder and Givens
  - Jacobi

III. Eigenvalues

- Reduction to tridiagonal form (Lanczos)
- Hessenberg reduction
- Algorithms:
  - power method and Rayleigh iteration
  - QR

IV. Model Problems and Applications

- Solution of Laplace/Poisson PDEs on a rectangle
- Compression of digital images