Additional problems on Ch 6, taken from S. Leon, Linear Algebra with Applications, 5th ed, Prentice-Hall, 1998

3. For each of the given vectors $\mathbf{x}$, find a Householder transformation such that $H\mathbf{x} = \alpha \mathbf{e}_1$, where $\alpha = \|\mathbf{x}\|_2$.

(a) $\mathbf{x} = (8, -1, -4)^T$ **(b)** $\mathbf{x} = (6, 2, 3)^T$ **(c)** $\mathbf{x} = (7, 4, -4)^T$

4. For each of the following, find a Householder transformation that zeros out the last two coordinates of the vector.

(a) $\mathbf{x} = (5, 8, 4, 1)^T$ **(b)** $\mathbf{x} = (4, -3, -2, -1, 2)^T$

6. Given

$$A = \begin{pmatrix} 1 & 2 & -4 \\ 2 & 6 & 7 \\ -2 & 1 & 8 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 9 \\ 9 \\ -3 \end{pmatrix}$$

(a) Use Householder transformations to transform A into an upper triangular matrix R. Also transform the vector $\mathbf{b}$, that is, compute $\mathbf{b}^{(1)} = H_2 H_1 \mathbf{b}$.

(b) Solve $R\mathbf{x} = \mathbf{b}^{(1)}$ for $\mathbf{x}$ and check your answer by computing the residual $\mathbf{b} - A\mathbf{x}$.

9. Let $H_k = I - 2\mathbf{u}\mathbf{u}^T$ be a Householder transformation with

$$\mathbf{u} = (0, \ldots, 0, u_k, u_{k+1}, \ldots, u_n)^T$$

Let $\mathbf{b} \in R^n$ and let A be an $n \times n$ matrix. How many additions and multiplications are necessary to compute (a) $H_k\mathbf{b}$; (b) $H_k A$?