

38.2. Suppose A is a real symmetric 805×805 matrix with eigenvalues $1.00, 1.01, 1.02, \dots, 8.98, 8.99, 9.00$ and also $10, 12, 16, 24$. How many steps of the conjugate gradient iteration must you take to be sure of reducing the initial error $\|e_0\|_A$ by a factor of 10^6 ?

38.3. The conjugate gradient is applied to a symmetric positive definite matrix A with the result $\|e_0\|_A = 1$, $\|e_{10}\|_A = 2 \times 2^{-10}$. Based solely on this data,

(a) What bound can you give on $\kappa(A)$?

(b) What bound can you give on $\|e_{20}\|_A$?

38.4. Suppose A is a dense symmetric positive definite 1000×1000 matrix with $\kappa(A) = 100$. Estimate roughly how many flops are required to solve $Ax = b$ to ten-digit accuracy by (a) Cholesky factorization, (b) Richardson iteration with the optimal parameter α (Exercise 35.3), and (c) CG.