TW793: Assignment 3 — 1 September

Assignment Guidelines

- Write a short report to illustrate your work. Explore and include interesting results (necessary to achieve $> 90\%$) and use full sentences to explain your reasoning.
- Reports may be handed in during the lecture on the due date.
- Feel free to discuss the work amongst one another, but write your own report and code.
- Working code has to be provided via e-mail, as an archive on the web or in an online code repository such as GitHub or BitBucket. You may use any open source language, such as Python, Octave, C#, etc.

Problem 1: Properties of the Fourier transform

The image g is a shifted version of f such that

$$g(x, y) = f(x - x_0, y - y_0).$$

The corresponding Fourier transforms are related by

$$G(u, v) = e^{-j2\pi(ux_0+vy_0)}F(u, v),$$

so that the cross-power spectrum becomes

$$\frac{F(u, v)G^*(u, v)}{|F(u, v)G(u, v)|} = e^{j2\pi(ux_0+vy_0)}.$$

(a) Show that the inverse Fourier transform of the cross-power spectrum is an impulse in the time domain. Where is this pulse located?

(b) Utilising this property, construct a routine that can automatically determine the shift between two images. (Use the provided test image to verify your results.)

Problem 2: Correlation using the FFT

Recall that convolution in the time domain is equivalent to multiplication in the frequency domain.

(a) Implement *correlation* using the fast Fourier transform (use the built-in `fft` or `fft2` function). Remember to pad your input signals appropriately! What happens if you don't? [HINT: for inputs of size $M \times N$ and $P \times Q$, pad to $M + P - 1 \times N + Q - 1$]

(b) For masks of different sizes, compare the execution speed of your new implementation with the one from the previous assignment.

Problem 3: Fourier-domain filters

Consider this heavily corrupted image of the moon landing:

(a) Compute and display the Fourier transform of the image. What do you observe?

(b) Make an educated guess as to the unwanted frequencies, and design a notch filter to remove the periodic noise.